Session:

A Multi-Scale Gas-Kinetic Scheme for Turbulence Simulation

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Turbulence is a typical multiscale problem and thus difficult for modelling and simulation. A multiscale method is of great advantage in efficiency in turbulence simulation, which can achieve smooth transition between different physical models according to different computational cell size. The hybrid RANS/LES method is such a multiscale one and attracts more and more researchers[1]. However, improper transition between different scales may lead to crucial problems, such as modelled stress depletion and logarithmic-layer mismatch. In recent years, the direct modelling method is proposed based on gas-kinetic theory and shows good performance in rarefied flows[2]. The success derives from the cross-scale time evolution solution from the model Boltzmann equation, such as BGK equation. In the present study, a new multiscale gas-kinetic method is developed for turbulence simulation.

Based on the “mixing-time” concept, Chen expanded the BGK equation to describe turbulence with the corresponding model of distribution function and relaxation time [3],

$$\frac{\partial f}{\partial t}+u_i\frac{\partial f}{\partial x_i}=\frac{g-f}{\tau+\tau_t}. \tag{1}$$

With the first order Chapman-Enskog expansion, the cross-scale time evolution solution of the above equation can be got as,

$$\begin{aligned} f\left(\mathbf{x}_{i+1/2,j,k},t,\mathbf{u},\xi\right)=&\frac{1}{\left(\tau+\tau_t\right)}\int_0^t\left(1-\tau\left(\partial_t+\mathbf{u}\cdot\nabla\right)\right)\tilde{g}\,\mathrm{e}^{-(t-t')/(\tau+\tau_t)}\,\mathrm{d}t' \\ &+\mathrm{e}^{-t/(\tau+\tau_t)}\left(1-\tau\left(\partial_t+\mathbf{u}\cdot\nabla\right)\right)g_0 \\ &-\frac{1}{\left(\tau+\tau_t\right)}\int_0^t\tau_t\left(\partial_t+\mathbf{u}\cdot\nabla\right)\tilde{g}\,\mathrm{e}^{-(t-t')/(\tau+\tau_t)}\,\mathrm{d}t' \\ &-\mathrm{e}^{-t/(\tau+\tau_t)}\,\tau_t\left(\partial_t+\mathbf{u}\cdot\nabla\right)g_0 \end{aligned}, \tag{2}$$

Session:

in which the terms contain g_0 and $\tilde{g}$ describe fluctuations for different scales respectively, and the last two terms are non-equilibrium effect controlled by turbulence relaxation time. In the present study the relaxation between different scales is determined by the time ratio $t/(\tau+\tau_t)$ and is modelled by the scale ratio of different turbulence model. S-A RANS model and Vreman-SM LES model[4] are considered. The developed method is named as multiscale gas-kinetic scheme (MS-GKS).

Several typical turbulent flows are simulated with MS-GKS, including channel flows and flows around a cylinder/periodic hills. The results are comparable with those from exiting hybrid method, such as DDES and IDDES, or even better (as shown in Fig.1). These preliminary tests show good potential of the present proposed MS-GKS in the turbulence simulation with high efficiency.

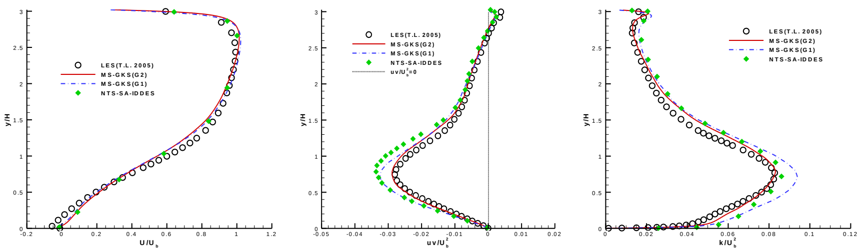

Fig. 1 Mean streamwise velocities, resolved shear stresses and kinetic energy at position x/H=4 computed by MS-GKS and compared with SA-IDDES in the flow past periodic hills with Re_H=10600.

References

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