Session:

A control strategy of the actual resolution in hybrid RANS/LES methods

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Over the past decade, hybrid RANS/LES methods (HRLM) have gained importance and have motivated a large research effort, seeking a compromise between the prediction capabilities of LES and the low cost of RANS computations. Among others, PITM and PANS can be mentioned, which are continuous hybrid methods, since they continuously transition from RANS to LES. They have another common feature: the use, as key parameter of the *energy ratio* (called hereafter r), which is the ratio of the modeled turbulent kinetic energy k_m to the total turbulent kinetic energy $k_{tot} = k_m + k_{res}$. The use of r distinguishes PITM, PANS and "Equivalent DES" from other HRLM (like DES for instance) which explicitely use the grid size Δ as key parameter for the RANS/LES transition.

However, during a computation, the *observed* ratio r^o does not match the *targetted* ratio r^t, prescribed by the user. This discrepancy is due to (i) the complex dynamics of turbulent flows, intrisically accounted for in the momentum equations, and (ii) the numerical method. This problem was already raised by [2], with an attempt of circumventing it.

The aim of the present work is to derive an efficient strategy to control the energy ratio in hybrid RANS/LES simulations. The derivation is done with the constraint of not changing the total turbulent kinetic energy (called hereafter k or k_{tot}). For the sake of clarity, we consider here the use of a first-order $k_{SFS} - \varepsilon_{SFS}$ subfilter closures.

The proposed strategy works as follows: (i) an extra "production" term is added in the transport equation for k_{SFS}, based on elastic spring forces, (ii) an extra force is added in the momentum equation, and it is calibrated in such a way that the overall turbulent kinetic energy k remains unchanged by the aforementioned extra-term. The basic motivation is to drive the *observed* energy ratio r^o to the level r^t that is *targetted* by the user. This is a control problem, and the hypothesis made here, is that a proportional "regulation" may be used in the transport equation for k_{SFS}, such that the mean production is:

$$\overline{P^{c,SFS}} = \frac{1}{\tau_R}\left(k_m^t - k_m^o\right)\ ,\ \text{with } k_m = \overline{k_{SFS}} \tag{1}$$

where τ_R is a relaxation time and the superscripts t and o respectively stand for targetted and observed. Using the definitions of r and k_m, one can easily show that the term above yields:

$$\boxed{P^{c,SFS} = \frac{k_{SFS}}{\tau_R}\left(\frac{r^t - r^o}{r^o}\right)}\ . \tag{2}$$

Then, in order to conserve the total turbulent kinetic energy conservation, the transport equation for $k_{res} = \overline{u_i'u_i'}/2$ must contain a compensating term. It is easy to show that this extra term should be $-\overline{P^{c,SFS}}$. One can show that adding an extra force vector $\mathbf{f^c}$ in the momentum equation, results in an extra term appearing in the transport equation for k_{res}, which we will call $\overline{P^{c,res}}$, such that:

$$\overline{P_{ij}^{c,res}} = \overline{f_i'^c u_j' + f_j'^c u_i'}\ , \tag{3}$$

where $\mathbf{f'^c} = \mathbf{f^c} - \overline{\mathbf{f^c}}$. Since we focus here on k_{res}, we only need the trace of (3): $\overline{P^{c,res}} = \overline{f_i'^c u_i'}$. Now, to ensure the conservation of the total turbulent kinetic energy, we must impose:

$$\overline{P^{c,res}} = -\overline{P^{c,SFS}}\ \Leftrightarrow\ \overline{f_i'^c u_i'} = -\frac{k_m}{\tau_R}\left(\frac{r^t - r^o}{r^o}\right)\ . \tag{4}$$

Averaged quantities are used in the relationship above, meaning that there is no unique solution $\mathbf{f^c}$. Thus, we use a linear forcing term [4], i.e. $f_i^c = Au_i'$, where A is a scalar. This formulation has a zero average, such that it does not change the mean velocity. The extra term is formulated as

$$\overline{f_i'^c u_i'} = A\overline{u_i' u_i'} = 2A\,\underbrace{k\,(1-r^o)}_{k_{res}} = 2A\frac{k_m}{r^o}\,(1-r^o)\ . \qquad (5)$$

Identifying (4) and (5), we obtain:

$$A = \frac{1}{2\tau_R}\left(\frac{r^o - r^t}{1-r^o}\right) \Leftrightarrow \boxed{f_i^c = \frac{1}{2\tau_R}\left(\frac{r^o - r^t}{1-r^o}\right)u_i'}\ . \qquad (6)$$

The targeted energy ratio, r^t, is computed by inverting Eq. 23 in [3], i.e.

$$r^t = 1 - (\psi - 1)/(c_{\varepsilon 2} - c_{\varepsilon 1}), \quad \psi = \max\left(1, L_t/(C_{DES}\Delta_{max})\right), \quad L_t = k_m^{3/2}/\varepsilon, \quad C_{DES} = 0.65 \qquad (7)$$

It is interesting to notice that the expression (6) is consistent with the isotropic forcing term proposed by [1]. The relaxation time τ_R must be arbitrarily set.

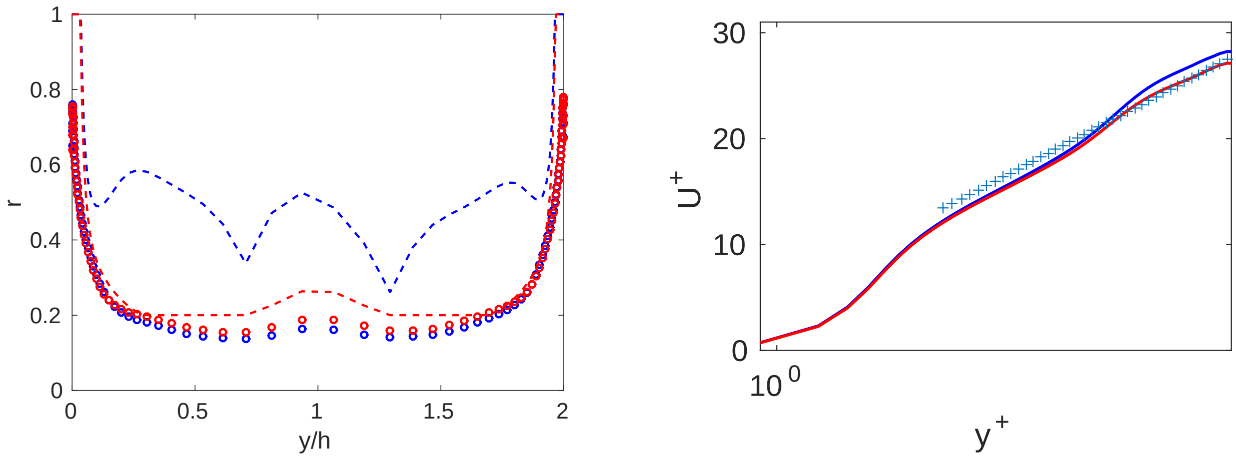

Figure 1: Comparison of PANS simulations of fully developped turbulent channel flow at $Re_\tau = 8000$, with (red) and without (blue) extra-terms given by Eqs. (2) and (6). Left: r fields (lines: r^t, circles: r^o), right: mean streamwise velocity.

Preliminary results were obtained on fully developped turbulent channel flow at $Re_\tau = 8000$, using an in-house finite volume code. The relaxation time τ_R is set to $h/u_\tau = 1$, where h is the half height of the channel and u_τ the friction velocity. Fig. 1a compares fields of r in the normalwise direction, and shows that the extra terms indeed allow for a better global convergence of r^o towards r^t. Moreover, Fig. 1b shows that the prediction of the mean streamwise velocity is improved when using the present control strategy, since the log-law is achieved at the centre of the channel. The final paper will contain more detailed results and discussion.

References

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