Consistent strain/stress lag eddy-viscosity model for hybrid RANS/LES

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Abstract The paper presents a comparison of several hybridization between RANS and LES, and their potential and limitations in an industrial context. The first part of the paper addresses an often eluded problem in the hybrid RANS/LES community: the importance of the baseline statistical model. A new robust low-Reynolds number eddy-viscosity model, derived from a Reynolds-stress model and accounting for the lag between stress and strain is extended to Detached Eddy Simulation (DES), and compared with the most widely used DES models.

The potential of two recently proposed hybridization of RANS and LES models is then discussed. The first one is based on a dual-mesh approach, where statistical and scale-resolving simulations are performed on separate grids, with drifts terms allowing to recover the most accurate solution on each grid. The current dual-mesh formulation is then compared with a consistent hybrid filtering combining RANS and LES.

1 Introduction

The generic use of hybrid turbulence models, blending statistical and scale-resolving approaches, in mainstream industrial applications, has been slow, despite being available in commercial codes for more than a decade. By far the most commong hybrid model is the Detached Eddy Simulation (DES), in few variants [4], but other methods have also found a limited success for some specific applications. Despite undeniable advantages over statistical RANS model, there are important limitation to overcome for a wider use of hybrid models in industry.

The first one is that the higher cost associated with running unsteady flow simulations is not always justified, especially if the goal is to obtain simple, time-averaged,

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statistics, which can be readily obtained using RANS: this is because the potential increase in accuracy only matters if the solution is robust. Convergence is however harder to quantify for scale-resolving simulations, there are are still uncertainties about the effect of initial conditions on long-term statistics, and results tend to be more mesh- (and sometime user)-dependent, adding to the overall cost of any industrial study (repetitiveness of results). For seamless hybrid methods such as DES, there are also uncertainties with the *grey-area* region, typically associated with a smooth blending function, and with a delayed response of all transported quantities. All of theses parameters lead to uncertainty in the flow analysis. The second problem, also often overlook in many publications, is that the most common hybrid models also lack generality for geometrically complex flows or when additional physical problems (multiphase, combustion, heat transfer) are added. This lack of generality is usually recognized for RANS models, for which specific modelling techniques have been available for many years, but are usually not being available or extensible to hybrid models.

The aim of this paper is to propose an alternative formulation to the most common DES model, addressing in particular the problem of RANS-to-LES transition. We then look at two new RANS/LES hybridization formulation that have been proposed recently: the consistent dual-mesh framework [6], recently improved by [7], and the properties of an hybrid RANS/LES filter and its relationship to current DES methods. Both methods are attractive in an industrial context because they are general methods allowing for a blending between any RANS model with any LES model.

2 Elliptic-blending Lag DES model

One of the main deficiency of eddy-viscosity models is that the Boussinesq approximation does not account for the lag between strain-rate and stress tensors. With the elliptic-blending lag (EBL) model, we extend the concept of introducing local anisotropic effect, as is done in the near-wall region with the $v^2 - f$ or elliptic-blending models, into the outer region as well. Noting that the exact production of turbulent kinetic energy is $P = -a_{ij}S_{ij}$, where a_{ij} is the anisotropy tensor and S_{ij} the strain-rate tensor, that in the eddy-viscosity framework $P = \nu_t S^2$, and using the classical definition of the eddy-viscosity of the elliptic-blending model ($\nu_t = C_\mu \varphi k \tau$), we define φ as

$$\varphi = -\frac{a_{ij}S_{ij}}{S^2}\frac{1}{C_\mu \tau} \qquad (1)$$

For simplicity, we take $\tau = k/\varepsilon$. A transport equation for φ can be derived using simply the transport equations for k, ε and a transport equation for a_{ij}, or equivalently, he transport equation for the Reynolds-stress $\overline{u_i u_j}$. The elliptic-blending Reynolds-stress model [2] is used as a baseline for the derivation. The resulting transport equation is given, exactly, by

$$\frac{D\varphi}{Dt} = \alpha^3 \left(\frac{C_{P3}}{\tau} + C_{P2}\varphi S + \frac{\tau_s^2}{\tau} \left(C_4^* a_{ik} S_{kj} - C_5^* a_{ik} W_{kj} \right) S_{ij} \right) - \left(1 - \alpha^3\right) C_w^* \frac{\varphi}{\tau}$$
$$- \alpha^3 \left(\widetilde{C}_1 + C_1^* \frac{P}{\varepsilon} \right) \frac{\varphi}{\tau} - C_{P1} \frac{\varphi P}{k} + \frac{\partial}{\partial x_j} \left[\left(\frac{\nu}{2} + \frac{\nu_t}{\sigma_k} \right) \frac{\partial \varphi}{\partial x_j} \right] \tag{2}$$

All the constants are calculated from the baseline EB-RSM [2], and do not need recalibration. A simplified expression for the anisotropy tensor a_{ij} is also derived specifically from the EB-RSM. The resulting linear model is supplemented with a robust universal wall-treatment and was tested on a large number of flow configurations, from simple channel flow to complex car geometries. The result is a model that behaves in many cases like an EB-RSM, but with the added robustness of being a simple eddy-viscosity model. It outperformed the most popular RANS models for separation prediction, and like the Reynolds-stress model, it is also intrinsically sensitive to rotation and curvature effects.

The EBL model has been extended to DES, using the same principle as that used for the IDDES [4]. A number of modifications to the IDDES model were however introduced: first, the blending parameter allows for a stronger near-wall shielding using the elliptic-blending parameter α. The other modifications were proposed by [5]: the length-scale now accounts for the log-layer mismatch, and the clipped length-scale is applied directly to the eddy-viscosity, providing a quicker transition from

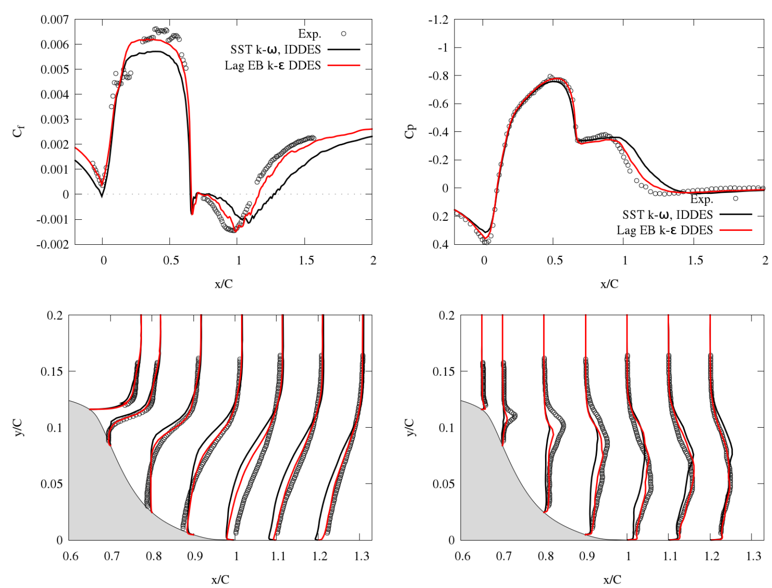

Fig. 1 Comparison of (from left to right, and top to bottom), the skin-friction coefficient, the pressure coefficient, the streamwise velocity and $\overline{uu}_{\text{tot}}/U_\infty^2$, between the SST $k-\omega$ IDDES and the Lag EB $k-\varepsilon$ DDES model. The velocity fluctuations include both modelled and resolved contributions.

RANS to LES, the formulation degenerating quicker into an exact Smagorinsky subgrid-scale model. The EBL-DES model was validated on a number of canonical cases, and results obtained on the 2D hump [3], a test case used in the EU project Go-4Hybrid, are shown as an exemple on Fig. 1.

3 Consistent RANS/LES framework

In recent years, alternative hybridization methods to DES have been proposed, and some of them have shown promising results, but mostly on simple cases. They thus remain far from being usable in an industrial context, but could potentially be used, if a number of points are addressed.

Noticing that in most industrial flow configuration, RANS is already providing good results, and that scale-resolving simulation is only needed in a small portion of the flow domain, where RANS shows well-known deficiencies, [6] have proposed a consistent dual-mesh formulation, wher the RANS is solved over the entire domain, and the LES mesh covers only the region where it is relevant (also slightly upstream and downstream). This method departs from other embedded LES/RANS methods for its use of two different grids, and for its use of a smooth interface. The main element of the model is the introduction of drift terms in the two sets of momentum equations, and turbulent transport equations, which force the flow statistics towards the most relevant formulation at a particular point in space. The drift terms are of the form [7]:

$$Q_i^L = (1-\beta)\,\frac{1}{\tau_d}\left[\left(U_i - \widehat{\tilde{u}}_i\right) + \frac{1}{C_d}\frac{k^L - k}{k^L + k}\left(\widehat{\tilde{u}}_i - \tilde{u}_i\right)\right] \tag{3}$$

$$Q_i^R = \beta\,\frac{1}{\tau_d}\left(\widehat{\tilde{u}}_i - U_i\right), \qquad Q_k = \beta\,\frac{1}{C_d \tau_d}\left(k^L - k\right) \tag{4}$$

where U_i is the average RANS velocity field, $\tilde{u}_i$ is the filtered LES velocity, and $\widehat{\tilde{u}}_i$ is the exponentially-weighted average (EWA) filtered LES velocity. k is the turbulent kinetic-energy, computed from the RANS solution, k_L is the EWA turbulent kinetic energy, and τ_d is a relaxation time-scale that needs to be defined. In regions where the RANS is trusted, the drift terms forces the EWA LES field towards the RANS field, and vice-versa, where the LES is well-resolved. Results obtained with an without the dual-mesh approach are shown on Fig. 2 for the periodic hill [8]. The LES alone is very under-resolved and does not provide satisfactory results, while the RANS performs reasonably well. When the drift terms are added (dashed lines), both RANS and time-averaged LES are consistent with each other, and the overall results shows a marked improvement over the individual methods. Although results are promising on canonical cases, the time-scale τ_d must be defined on the case-by-case basis, unlikely to be suitable for industrial applications. There are also uncertainties as to the definition of the EWA time-scale. The dual-mesh formulation [7] will be revisited in the light of the consistent hybrid filter model [9].

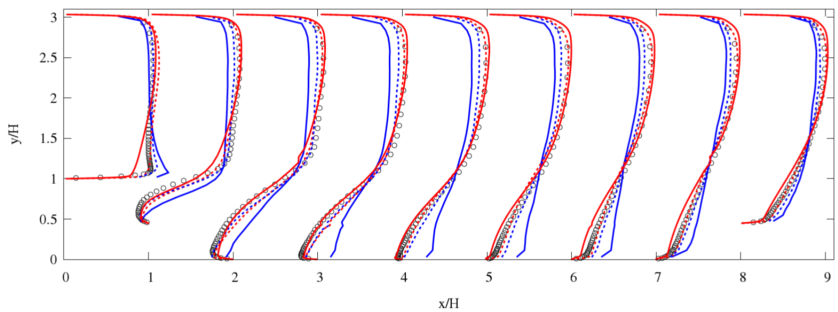

Fig. 2 Periodic hill flow: (top) meshes used for the RANS solution (red) and the LES (blue). Bottom plot: averaged RANS (solide blue line) and LES field without drifts terms, and with drift terms (dashed lines).

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