Progress in Hybrid Temporal LES

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In order to favour the modelling of the subgrid stresses in continuous Hybrid RANS/LES methods, the comparison of the solutions with experimental or DNS databases, and eventually the understanding of the phenomenology observed in the resolved motion, defining a rigorous formalism for such methods is highly desirable. Empirical methods to bridge RANS and LES, based on the modification of the scales used in a RANS model in order to reduce the level of modelled stresses in specific regions and enforce a migration towards a LES behaviour, suffer from the fact that RANS and LES are based on generally inconsistent operators, statistical averaging and spatial filtering, respectively.

In order to investigate the possibility of building a consistent formalism for hybrid RANS/LES, it is convenient to try to express averaging operators, following Kampe de Feriet and Betchov (1951), as the limiting case of a general spatio-temporal filtering operator of the form

$$\overline{u}(\mathbf{x},t) = \int\int G(\mathbf{x},\mathbf{x}',t,t')u(\mathbf{x}',t')\mathrm{d}\mathbf{x}'\mathrm{d}t',$$

where the filter kernel G is parametrized by one or several filter widths. A consistent hybrid RANS/LES formulation can be obtained if the filter kernel G goes to a usual filter for small filter widths (LES mode) and to the statistical average for infinite filter widths (RANS mode). In the particular case of homogeneous turbulence, statistical averaging is equivalent to spatial averaging, and Chaouat & Schiestel (2005) have proposed the PITM approach, based on standard spatial filtering, which is a subset of the general filtering above. A more general approach, applicable to inhomogeneous turbulence, is the additive filter of Germano (2004). For instance, in the case of inhomogeneous, stationary turbulence, for which statistical averaging is equivalent to long-time averaging, the additive filter corresponds to a spatio-temporal filter kernel of the form

$$G(\mathbf{x},\mathbf{x}',t,t') = \beta G_S(\mathbf{x},\mathbf{x}')\delta(t'-t) + (1-\beta)\delta(\mathbf{x}'-\mathbf{x})G_T(t,t'),$$

where G_S is a spatial kernel, δ the Dirac delta function, β a blending parameter and G_T a temporal kernel with a sufficiently large filter width.

For inhomogeneous, stationary turbulence, since any temporal filter goes to the statistical average in the limit of an infinite filter width, it is tempting to simplify the formalism by using a standard, Eulerian temporal filter

$$G(\mathbf{x},\mathbf{x}',t,t') = \delta(\mathbf{x}'-\mathbf{x})G_T(t,t').$$

However, as shown by Speziale (1987), Eulerian temporal filtering does not satisfy Galilean invariance, such that it is not an acceptable formalism. In order to remedy this problem, we introduce the generalized temporal filter

$$G(\mathbf{x},\mathbf{x}',t,t') = \delta(\mathbf{x}'-\xi(\mathbf{x},t,t'))G_T(t,t'),$$

which is characterized by a moving application point. A particular case is the Lagrangian filter, for which

$$\xi(\mathbf{x}, t, t') = \chi(\mathbf{x}, t'),$$

where χ denotes the fluid particle trajectory. In order to avoid the complexity of Lagrangian filtering, while preserving the Galilean invariance, a *uniform motion* of the application point is used instead:

$$\xi(\mathbf{x}, t, t') = \mathbf{x} - U_{\mathrm{ref}}(t' - t),$$

where U_{ref} is an arbitrary velocity constant in space and time, and the filter is refered to as the *uniform temporal filter*. This filter is thus a Galilean invariant temporal filter. In the particular case of stationary turbulence, it can be easily seen that the filtered quantities go to statistically averaged quantities in the limit of infinite filter width, using the reference velocity $U_{\mathrm{ref}} = 0$ (Kampe de Feriet, 1957). Within this formalism, the equations of motion for the filtered variables can be derived that are formally similar to the standard LES or RANS equations, as soon as the subfilter stresses are expressed in terms of generalized central moments (Germano, 1992). This form invariance constitutes a solid foundation for the development of continuous hybrid RANS/LES models.

The presence in the equations of the subfilter stresses (SFS), due to temporally filtered-out scales, lead to a closure problem similar to the case of standard LES, although the variables have a different definition. This issue was addressed in the framework of pure Temporal LES (i.e., with a frequency cutoff in the inertial zone of the spectrum) by Pruett (2000), Pruett *et al*. (2003) and Tejada-Martinez *et al*. (2007). For hybrid RANS/LES, as shown by Fadai-Ghotbi *et al.* (2010), the approach proposed by Chaouat and Schiestel (2005) in the framework of spatial filtering can be transposed to uniform temporal filtering, leading to the so-called Temporal Partially Integrated Transport Method (TPITM). This approach results in models based on transport equations for the subfilter energy or stress and for the dissipation rate analogous to the standard RANS models, but with a variable $C^*_{\varepsilon 2}$ coefficient in the dissipation equation, function of the ratio of modelled to total turbulent energy $r = k_{\mathrm{sfs}}/k$ as

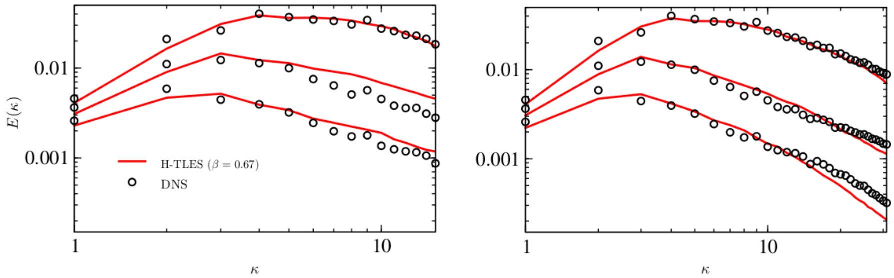
$$C^*_{\varepsilon 2} = C_{\varepsilon 1} + r(C_{\varepsilon 2} - C_{\varepsilon 1}),$$

such that the RANS model ($C^*_{\varepsilon 2} = C_{\varepsilon 2}$) is recovered for $r = 1$ and the LES mode is activated for $r < 1$. The value of the ratio r can be related to the cutoff frequency of the temporal filter ω_c using the assumption of an equilibrium Eulerian spectrum (Tennekes, 1975), leading to

$$r = \min\left(1; \frac{1}{\beta}\left(\frac{U_s}{\sqrt{k}}\right)^{2/3}\left(\omega_c \frac{k}{\varepsilon}\right)^{-2/3}\right).$$

Similar to the case of the cutoff wavenumber in standard LES, the cutoff frequency can be related to the highest observable frequency, linked to the time step dt and the grid step Δ by

$$\omega_c = \min\left(\frac{\pi}{dt}; \frac{U_s \pi}{\Delta}\right).$$



(a) 32^3 grid (b) 64^3 grid

Fig. 1: Calibration of the HTLES approach in homogeneous isotropic turbulence

U_s denotes the sweeping velocity (Tennekes,1975), i.e., the maximum velocity at which the small scales are convected by the combination of the mean flow and the large scales, $U_s=U+u$, with U the mean velocity magnitude and $u= \gamma\, k^{1/2}$.

Applications of this method (Fadai-Ghotbi *et al.*, 2010) showed the difficulty to sustain resolved turbulent fluctuations during the computation of some flows, in particular flows that are not dominated by the Kelvin-Helmholtz instability, and the necessity of introducing a dynamic procedure to avoid a pseudo-laminarization of the computed flow. This problem can be attributed to the fact that the migration from RANS to LES, or in other words, the level of modelled stresses, is only indirectly controlled via the dissipation equation. In order to address this issue, Friess *et al.* (2015) proposed to derive an equivalent approach, in which this control is performed via a direct modification of the dissipation term in the turbulent energy or stresses, in the spirit of DES. Such an approach can be obtained by defining the H-*equivalence criterion*, where H stands for *Hybrid.* Postulating that *Two hybrid approaches based on the same closure, but using a different method of control of the energy partition, yield similar low-order statistics of the resolved velocity fields provided that they yield the same level of subfilter energy* (Friess *et al*., 2015), two approaches are said H-equivalent if they lead to the same partition of energy for a particular situation and tend to the same RANS model for large filter widths.

Using a perturbation method, i.e., based on infinitesimal modifications of the system of equations, it can be shown that, e.g. in inhomogeneous, fully developed flows, the approach based on a modified transport equation for the subfilter energy k_{sfs} of the form

$$\frac{\mathrm{d}k_{\text{sfs}}}{\mathrm{d}t} = P_{\text{sfs}} - \frac{k_{\text{sfs}}}{T} + D_{\text{sfs}},$$

can be H-equivalent to the TPITM model if the time scale is written as

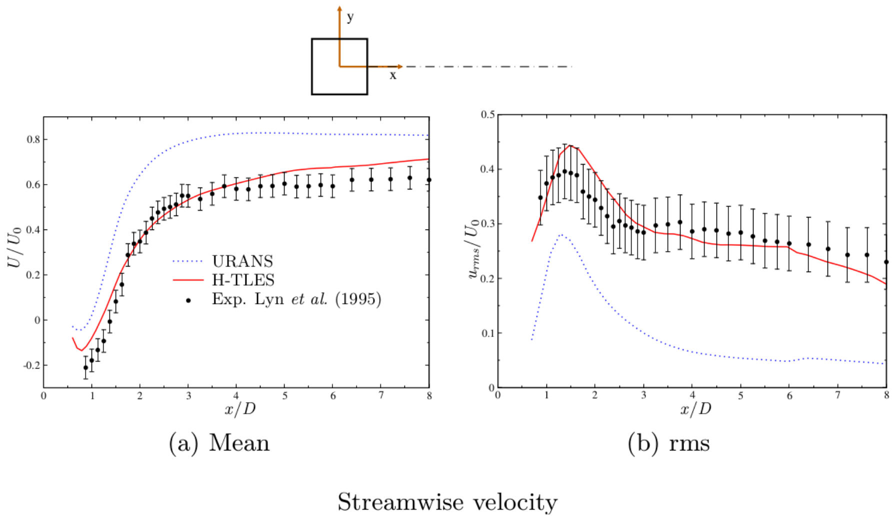


Fig. 2:Application of the HTLES approach in the case of the flow around a square cylinder

$$T = \frac{r}{1 + \left(\frac{C_{\varepsilon 2}}{C_{\varepsilon 1}} - 1\right)\left(1 - r^{C_{\varepsilon 1}/C_{\varepsilon 2}}\right)} \frac{k}{\varepsilon}.$$

The transport equation for k_{sfs} is formally identical to a standard RANS model equation, in which the production term P_{sfs} and the diffusion term D_{sfs} can have different expressions, depending on the model. The modified time scale enforces the LES mode for $r < 1$ by increasing the dissipation term, similar to DES, which is however based on a modified length scale. The present approach, based on uniform temporal filtering and a modification of the time scale in the transport equation for the subfilter energy, is called Hybrid Temporal LES (HTLES). It can be easily adapted to second moment closures, i.e., models based on transport equations for the subfilter stress tensor.

Tran *et al*. (2012) have calibrated the β coefficient that appears in the evaluation of r as a function of the cutoff frequency in the case homogeneous isotropic turbulence, in comparison with the DNS data of Wray (1998). Fig. 1 shows, for two grid refinements, that the correct level of subfilter dissipation is provided by the model in the LES mode for $\beta = 0.67$. The HTLES approach is applied here to the k-ω-SST model (Menter, 1994). Fig. 2 shows results obtained for the flows around a square cylinder, in comparison with the experimental data of Lyn et al. (1995).

It is interesting to note that, if the cutoff frequency of the temporal filter is imposed as the characteristic frequency of the large scales $\omega_c = \varepsilon/k$, the dissipation term k_{sfs}/T in the transport equation for k_{sfs} is equal to ε, such that the URANS equations are recovered (i.e., including the time derivative). Therefore, URANS, which suffers from a lack of clear definition in general situations, can be regarded as a temporally-filtered approach in which the temporal filter width is chosen as the eddy-turnover time of the energetic structures of the flow. However, it can be seen in Fig. 2 that the URANS approach, although it is able to reproduce the vortex shedding with the correct Strouhal number, is not able to reproduce the amplitude of the vortices and the related deficit of streamwise velocity. The HTLES approach, using the same closure, by reducing the amount of modelled stresses in the momentum equation, makes possible the computation of a wider range of turbulent structures and drastically improve the results.

References

Chaouat, B. & Schiestel, R. (2005), 'A new partially integrated transport model for subgrid-scale stresses and dissipation rate for turbulent developing flows', *Phys. Fluids* 17(065106), 1-19.

Friess, C.; Manceau, R. & Gatski, T. (2015), 'Toward an equivalence criterion for hybrid RANS/LES methods', *Comput. Fluids* 122, 233-246.

Germano, M. (1992), 'Turbulence: the filtering approach', *J. Fluid Mech.* 238, 325-336.

Germano, M. (2004), 'Properties of the hybrid RANS/LES filter', *Theor. Comput. Fluid Dyn.* 17(4), 225-231.

Kampé de Fériet, J. (1957), La notion de moyenne dans la théorie de la turbulence, *in* 'Rendiconti del seminario matematico dell'universita e del politecnico', Libreria editrice politecnica, Cesare Tamburini, Milano, , pp. 167-207.

Kampé de Fériet, J. & Betchov, R. (1951), 'Theoretical and experimental averages of turbulent functions', *Proc. K. Ned. Akad. Wet.* 53, 389-398.

Lyn, D.; Einav, S.; Rodi, W. & Park, J.-H. (1995), 'Laser-Doppler velocimetry study of ensemble-averaged characteristics of the turbulent near wake of a square cylinder', *J. Fluid Mech.* 304, 285-319.

Menter, F. R. (1994), 'Two-equation eddy-viscosity turbulence models for engineering applications', *AIAA J.* 32(8), 1598-1605.

Pruett, C. D. (2000), 'Eulerian time-domain filtering for spatial large-eddy simulation', *AIAA J.* 38(9), 1634-1642.

Pruett, C. D.; Gatski, T. B.; Grosch, C. E. & Thacker, W. D. (2003), 'The temporally filtered Navier-Stokes equations: Properties of the residual stress', *Phys. Fluids* 15(8), 2127-2140.

Speziale, C. G. (1987), 'On the decomposition of turbulent flow fields for the analysis of coherent structures', *Acta Mechanica* 70(1-4), 243-250.

Tejada-Martinez, A. E.; Grosch, C. E. & Gatski, T. B. (2007), 'Temporal Large eddy simulation of unstratified and stably stratified turbulent channel flows', *Int. J. Heat Fluid Fl.* 28, 1244-1261.

Tennekes, H. (1975), 'Eulerian and Lagrangian time microscales in isotropic turbulence', *J. Fluid Mech.* 67, 561-567.

Tran, T.; Manceau, R.; Perrin, R.; Borée, J. & Nguyen, A. (2012), A hybrid temporal LES approach. Application to flows around rectangular cylinders, *in* 'Proc. 9th ERCOFTAC Int. Symp. on Eng. Turb. Modelling and Measurements, Thessaloniki, Greece'.

Wray, A. (1998), *A selection of test cases for the validation of large-eddy simulations of*